#### Linear Programming Sample Problems

1. The Win Big Gambling Club promotes gambling junkets from a large Midwestern city to casinos in the Bahamas. The club has budgeted up to $8000 per week for local advertising, and the money is to be allocated among four media: TV, newspaper, radio, and/or direct mail. Win Big’s goal is to reach the largest audience of potential gamblers through the various media. The following table presents the number of potential gamblers reached by making use of an advertisement in each of the four media. It also provides figures on the cost per advertisement placed and the maximum number of ads that can be purchased per week.

|  |  |  |  |
| --- | --- | --- | --- |
| **Medium** | **Audience per Unit** | **Cost per Unit** | **Max Units per Week** |
| **TV** | 10000 | $1000 | 5 |
| Paper | 8500 | $750 | 10 |
| **Radio** | 3000 | $500 | 10 |
| **Mail** | 5000 | $400 | 5 |

Win Big’s contractual arrangements require that at least three radio spots be placed each week. To ensure a broad promotional campaign, the management also insists that no more than $2000 be spent on radio and TV combined every week.

1. Formulate and solve as a LP model.
2. How much better Win Big could do if upper management did away with the $2000 constraint on radio and TV?
3. The Piggly Wiggly hog farm can use its land either for raising hogs or for growing corn. The hog farm has a total of 1000 acres of land, and they need to decide what percentage should be allocated to raising hogs and what percentage to growing corn. Any corn produced can either be sold on the open market or used as hog feed. Suppose that they can raise 200 hogs on each acre of land. If they grow corn on the land, they can expect a yield of 175 bushels per acre. Suppose that each hog requires 20 bushels of feed to raise it up to the age at which it can be turned into bacon. Hog feed costs $7 per bushel, but corn can be substituted for hog feed on a 1-1 basis. That is, 1 bushel of hog food will replace 1 bushel of hog feed. The hog farm can sell its grown hogs for an average of $300 apiece. Costs of raising the hogs are, excluding feed costs, approximately $40 per hog. The hog farm can sell its corn on the open market for $5 per bushel instead of using it for hog feed. Costs of growing the corn amount to approximately $300 per acre. How much land should be devoted to corn and how much to hogs? Can you perform a sensitivity analysis on the open market price of corn to determine how high the price would have to go to entice the hog farm to devote all its land to raising corn for sale on the open market?

3. An investor wishes to invest $10,000 for the coming year to maximize her guaranteed (or “risk free”) return. She has identified that the market will be in one of four different states at the end of the year, and these states affect her investments in each of three possible stocks and a bond as shown in the table below. The table provides the current price of each possible instrument as well as projected year-end prices of each instrument under each of the 4 possible states.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Price | State 1 | State 2 | State 3 | State 4 |
| Stock A | $2.00 | $1.80 | $1.70 | $2.70 | $2.50 |
| Stock B | $2.50 | $2.25 | $3.50 | $2.00 | $2.70 |
| Stock C | $3.00 | $3.90 | $2.70 | $3.60 | $2.50 |
| Bond D | $1.00 | $1.04 | $1.04 | $1.04 | $1.04 |

Formulate her investment problem as a linear program and solve it using Excel. How much should she invest in each security?

1. Read the Serendipity Canners mini-case. Answer the following questions.
2. Why does Tucker state that the whole tomato production is limited to 800,000pounds (i.e., where does the number 800,000 come from?)
3. What is wrong with Cooper’s suggestion to use the entire crop for whole tomatoes?
4. How does Myers compute his tomato costs in Exhibit 3? How does he reach his conclusion that the company should use 2,000,000 pounds of “B” tomatoes for paste, the remaining 400,000 pounds of “B” tomatoes, and all of the “A” in juice? What is wrong with Myers’s reasoning? Hint: it IS wrong.
5. Without including the possibility of the additional purchases suggested by Gordon, formulate and solve as a linear program the problem of determining the optimal production mix for this year’s crop. Define your decision variables in terms of pounds of tomatoes. Express the objective function coefficients in dollars per pound. Be sure to include quality constraints.

5. The Three Princes of Serendip: An early application of linear programming

**The Three Princes of Serendip**

*by H. Walpole*

The three princes of Serendip

Went on a little trip.

They could not carry too much weight;

More than 300 pounds made them hesitate.

They planned to the ounce. When they returned to Ceylon

They discovered that their supplies were just about gone

When, what to their joy, Prince William found

A pile of coconuts on the ground.

“Each will bring 60 rupees,” said Prince Richard with a grin

As he almost tripped over a lion skin.

“Look out!” cried Prince Robert with glee

As he spied some more lion skins under a tree.

“These are worth even more - 300 rupees each

If we can just carry them down to the beach.”

Each skin weighed fifteen pounds and each coconut, five,

But they carried them all and made it alive.

The boat back to the island was very small

Fifteen cubic feet baggage capacity - that was all.

Each lion skin took up one cubic foot

While eight coconuts the same space took.

With everything stowed they headed to sea

And on the way calculated what their new wealth might be.

“Eureka!” cried Prince Robert, “Our worth is so great

That there’s no other way we could return in this state.

Any other skins or nuts which we might have brought

Would now have us poorer. And now I know what -

I’ll write my friend Horace in England, for surely

Only he can appreciate our serendipity.

Formulate and solve as a linear programming problem - How many nuts and skins?